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有源加载准光腔的并矢格林函数*

何宜军 杨铨让

(东南大学无线电工程系毫米波国家重点实验室，江苏，南京， 210018)

摘要： 由近轴近似得到了有源加载准光腔的矢量波函数，并由矢量波函数展开方法得到了有源加载情况下准光腔中的并矢格林函数．由此可以得到腔中电磁场的积分解．

关键词： 并矢格林函数，准光腔，有源加载．

引言

当已知某种结构的并矢格林函数后，即可求解有关这种结构的电磁场边值问题，所以并矢格林函数在电磁理论中十分有用．矩形和圆柱形波导以及谐振腔和空开放腔的并矢格林函数已经求得[1−4]．本文利用近轴近似方法得到了有源加载准光腔的并矢格林函数．

1 理论分析

图 1 为有源加载准光腔结构，它由两个相同的球面镜和一无限大介质平板组成，坐标原点在对称中心点．这种结构的并矢格林函数 $\overline{\overline{G}}$ 满足如下方程：

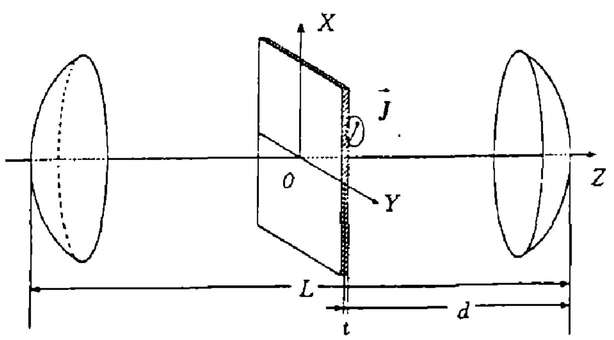

图 1 有源加载准光腔结构
Fig. 1 The structure of the active loaded quasi-optical cavity

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$$\nabla \times \nabla \times \overline{\overline{G}}(\overline{R}|\overline{R}') - k^2\overline{\overline{G}}(\overline{R}|\overline{R}') = \overline{\overline{I}}\delta(\overline{R} - \overline{R}'), \tag{1}$$

其中 $\overline{\overline{I}}$ 为单位并矢，上标 "=" 代表并矢. 在腔表面上有

$$\hat{n} \times \overline{\overline{G}}(\overline{R}|\overline{R}') = 0, \tag{2}$$

式 (2) 中 $\hat{n}$ 为腔法外向单位矢量.

矢量本征函数 $\overline{M}$ 和 $\overline{N}$ 分别定义为

$$\overline{M} = \nabla \times (\psi\hat{Z}), \tag{3}$$

$$\overline{N} = \frac{1}{k_\nu} \nabla \times \overline{M}, \tag{4}$$

其中 ψ 满足波动方程

$$\nabla^2\psi + k_\nu^2\psi = 0, \tag{5}$$

式 (5) 中 k_ν^2 为本征值. 利用近轴近似方法[5]，我们得到了两个区域中的标量波函数. 在区域 1 $(0 \le Z \le t)$ 中:

$$\begin{aligned}\psi_{mn} = {} & C_{mn}\frac{\omega_0}{\omega_1(Z)}H_m\left(\frac{\sqrt{2}}{\omega_1(Z)}x\right)H_n\left(\frac{\sqrt{2}}{\omega_1(Z)}y\right)\cdot\exp\left(-\frac{x^2+y^2}{\omega_1^2(Z)}\right)\\ & \cdot\exp\left\{-j\left[n_0k_\nu Z + \frac{n_0k_\nu}{2R_1(Z)}(x^2+y^2) - (m+n+1)\Phi_1(Z)\right]\right\},\end{aligned} \tag{6}$$

其中

$$\begin{cases} \omega_1^2(Z) = \omega_0^2\left(1 + \dfrac{Z^2}{n_0^2Z_0^2}\right), \\ R_1(Z) = Z + \dfrac{n_0^2Z_0^2}{Z}, \\ \Phi_1(Z) = \arctan\left(\dfrac{Z}{n_0Z_0}\right), \\ Z_0 = \dfrac{1}{2}k_\nu\omega_0^2. \end{cases} \tag{7}$$

在区域 2 $(0 \le Z \le t+d)$ 中:

$$\begin{aligned}\psi_{mn} = {} & C_{mn}\frac{\omega_0}{\omega_2(Z)}H_m\left(\frac{\sqrt{2}}{\omega_2(Z)}x\right)H_n\left(\frac{\sqrt{2}}{\omega_2(Z)}y\right)\cdot\exp\left(-\frac{x^2+y^2}{\omega_2^2(Z)}\right)\\ & \cdot\exp\left\{-j\left[k_\nu(Z-t') + \frac{k_\nu}{2R_2(Z)}(x^2+y^2) - (m+n+1)\Phi_2(Z)\right]\right\};\end{aligned} \tag{8}$$

其中

$$\begin{cases} \omega_2^2(Z) = \omega_0^2\left(1 + \dfrac{(Z-t^{'})^2}{Z_0^2}\right), \\ R_2(Z) = Z - t^{'} + \dfrac{Z_0^2}{Z-t^{'}}, \\ \varPhi_2(Z) = \arctan\left(\dfrac{Z-t^{'}}{Z_0}\right), \\ t^{'} = t\left(1 - \dfrac{1}{n_0}\right); \end{cases} \tag{9}$$

其中 n_0 为介质的折射率.

在频率较高时，由于波长很短，Z_0 很大，则式 (7) 和式 (9) 中的 ω_1, R_1, $\varPhi_1$, ω_2, R_2, $\varPhi_2$ 可近似为

$$\begin{aligned} &\omega_1(Z) \approx \omega_2(Z) \approx \omega_0, \\ &R_1(Z) \approx R_2(Z) \to \infty, \\ &\varPhi_1(Z) \approx \frac{Z}{n_0 Z_0}, \\ &\varPhi_2(Z) \approx \frac{Z-t^{'}}{Z_0}; \end{aligned}$$

此时腔内的驻波解为

$$\begin{aligned} \psi_{mn}^{e,0} = {}& C_{mn}^{e,0}\cdot H_m\left(\frac{\sqrt{2}}{\omega_0}x\right)H_n\left(\frac{\sqrt{2}}{\omega_0}y\right)\exp\left(-\frac{x^2+y^2}{\omega_0^2}\right) \\ &\cdot\begin{cases} \begin{matrix}\cos\\ \sin\end{matrix}\left[n_0 k_\nu Z - (m+n+1)\dfrac{Z}{n_0 Z_0}\right], & (0 \le Z \le t) \\ \begin{matrix}\cos\\ \sin\end{matrix}\left[k_\nu (Z-t^{'}) - (m+n+1)\dfrac{Z-t^{'}}{Z_0}\right]; & (t \le Z \le t+d) \end{cases} \end{aligned} \tag{10}$$

其中

$$k_\nu = (m+n+1)\cdot\frac{1}{Z_0} + \frac{q}{t+d-t^{'}}\pi, \qquad (q = 0, \pm 1, \pm 2, \cdots)$$

将式 (9) 的标量波函数代入式 (3) 和式 (4)，便可求得近似情况下的矢量波函数：

$$\begin{aligned} \overline{M}^{e,0} = {}& C_{mn}^{e,0}\left\{\left[\frac{2\sqrt{2}n}{\omega_0}H_{n-1}\left(\frac{\sqrt{2}}{\omega_0}y\right) - \frac{2y}{\omega_0^2}H_n\left(\frac{\sqrt{2}}{\omega}y\right)\right]\cdot H_m\left(\frac{\sqrt{2}}{\omega_0}x\right)\hat{x}\right. \\ &\left. - \left[\frac{2\sqrt{2}m}{\omega_0}H_{m-1}\left(\frac{\sqrt{2}}{\omega_0}x\right) - \frac{2x}{\omega_0^2}H_m\left(\frac{\sqrt{2}}{\omega_0}\right)\right]\cdot H_n\left(\frac{\sqrt{2}}{\omega_0}y\right)\hat{y}\right\}\cdot\exp\left(-\frac{x^2+y^2}{\omega_0^2}\right) \\ &\cdot\begin{cases} \begin{matrix}\cos\\ \sin\end{matrix}\left(n_0 k_\nu Z - (m+n+1)\dfrac{Z}{n_0 Z_0}\right), & (0 \le Z \le t) \\ \begin{matrix}\cos\\ \sin\end{matrix}\left(k_\nu (Z-t^{'}) - (m+n+1)\dfrac{Z-t^{'}}{Z_0}\right); & (t \le Z \le t+d) \end{cases} \end{aligned} \tag{11}$$

$$\overline{N}^{e,0}=\frac{C_{mn}^{e,0}}{k_\nu}\left\{\left[\frac{2\sqrt{2}m}{\omega_0}H_{m-1}\left(\frac{\sqrt{2}}{\omega_0}x\right)-\frac{2x}{\omega_0^2}H_m\left(\frac{\sqrt{2}}{\omega_0}x\right)\right]H_n\left(\frac{\sqrt{2}}{\omega_0}y\right)\hat{x}\right.$$

$$\left.+\left[\frac{2\sqrt{2}n}{\omega_0}H_{n-1}\left(\frac{\sqrt{2}}{\omega_0}y\right)-\frac{2y}{\omega_0^2}H_n\left(\frac{\sqrt{2}}{\omega_0}y\right)\right]\cdot H_m\left(\frac{\sqrt{2}}{\omega_0}x\right)\hat{y}\right\}\exp\left(-\frac{x^2+y^2}{\omega_0^2}\right)$$

$$\cdot\begin{cases}\begin{matrix}-\sin\\ \cos\end{matrix}\left[n_0k_\nu Z-(m+n+1)\frac{Z}{n_0Z_0}\right]\cdot\left[n_0k_\nu-(m+n+1)\frac{1}{n_0Z_0}\right]\\ \qquad\begin{matrix}-\sin\\ \cos\end{matrix}\left[k_\nu(Z-t')-(m+n+1)\frac{Z-t'}{Z_0}\right]\left[k_\nu-(m+n+1)\frac{1}{Z_0}\right]\end{cases}$$

$$+\hat{Z}\frac{C_{mn}^{e,0}}{k_\nu}H_m\left(\frac{\sqrt{2}}{\omega_0}x\right)H_n\left(\frac{\sqrt{2}}{\omega_0}y\right)\exp\left(-\frac{x^2+y^2}{\omega_0^2}\right)\times \tag{12}$$

$$\begin{cases}\begin{matrix}\cos\\ \sin\end{matrix}\left[n_0k_\nu Z-(m+n+1)\frac{Z}{n_0Z_0}\right]\\ \qquad\qquad\left[k_\nu^2-\left(n_0k_\nu-\frac{m+n+1}{n_0Z_0}\right)^2\right],\quad(0\le Z\le t)\\ \begin{matrix}\cos\\ \sin\end{matrix}\left[k_\nu(Z-t')-(m+n+1)\frac{Z-t'}{Z_0}\right]\\ \qquad\qquad\left[k_\nu^2-\left(k_\nu-\frac{m+n+1}{Z_0}\right)^2\right];\quad(t\le Z\le t+d)\end{cases}$$

由此很容易证明矢量波函数的正交性. 考虑到标量波函数的归一化系数

$$C_{mn}^{e^2}=C_{mn}^{0^2}=\frac{1}{m!\,n!\,\omega_0^2\pi\cdot 2^{m+n-3}\cdot L}, \tag{13}$$

于是，我们得到

$$\int\overline{M}_{mnq}^{e,0}\cdot\overline{M}_{m'n'q'}^{e,0}\mathrm{d}V$$

$$=\begin{cases}\frac{4(m+n+1)}{\omega_0^2}-\frac{4(m+n+1)^2}{k_\nu^2\omega_0^4}, & (m=m',\ n=n',\ q=q')\\ 0; & (\text{其它})\end{cases} \tag{14}$$

$$\int\overline{N}_{mnq}^{e,0}\cdot\overline{N}_{m'n'q'}^{e,0}\mathrm{d}V$$

$$=\begin{cases}\frac{4(m+n+1)}{\omega_0^2}-\frac{4(m+n+1)^2}{k_\nu^2\omega_0^4}, & (m=m',\ n=n',\ q=q')\\ 0; & (\text{其它})\end{cases} \tag{15}$$

对式 (1) 取旋度，则有

$$\nabla\times\nabla\times\nabla\times\overline{\overline{G}}(\overline{R}|\overline{R}')-k^2\nabla\times\overline{\overline{G}}(\overline{R}|\overline{R}')=\nabla\times[\overline{\overline{I}}\delta(\overline{R}-\overline{R}')], \tag{16}$$

因为 $\nabla \times [\bar{\bar{I}}\delta(\overline{R} - \overline{R}')]$ 是无散的，则它可由式 (3) 和式 (4) 决定的本征矢量波函数展开. 令

$$\nabla \times [\bar{\bar{I}}\delta(\overline{R} - \overline{R}')] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{q=0}^{\infty}(\overline{M}^{e}_{mnq}\overline{A}_{emnq} + \overline{N}^{o}_{mnq}\overline{B}_{0mnq}), \tag{17}$$

利用 $\overline{M}^{e}_{mnq}$ 和 $\overline{N}^{o}_{mnq}$ 的正交关系，则有

$$\overline{A}_{emnq} = \frac{1}{W_{eM}} \nabla' \times \overline{M}^{e'}_{mnq}, \tag{18}$$

$$\overline{B}_{omnq} = \frac{1}{W_{oN}} \nabla' \times \overline{N}^{o'}_{mnq}; \tag{19}$$

其中

$$W_{eM} = W_{oN} = \frac{2}{\omega_0^2}(m+n+1). \tag{20}$$

应用 Ohm–Rayleigh[1]方法可以求得

$$\nabla \times \bar{\bar{G}}(\overline{R}|\overline{R}') = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{q=0}^{\infty}\left[\frac{1}{\omega_{eM}(k_\nu^2 - k^2)}\right] \cdot \left[\overline{M}^{e}_{mnq} \nabla' \times \overline{M}^{e'}_{mnq} + \overline{N}^{o}_{mnq} \nabla' \times \overline{N}^{o'}_{mnq}\right], \tag{21}$$

由式 (1) 可得

$$\bar{\bar{G}}(\overline{R}|\overline{R}') = \frac{1}{k^2}[\nabla \times \nabla \times \bar{\bar{G}}(\overline{R}|\overline{R}') - \bar{\bar{I}}\delta(\overline{R}|\overline{R}')], \tag{22}$$

对式 (21) 取旋度，且考虑到式 (4) 及其对应形式

$$\overline{M} = \frac{1}{k_\nu} \nabla \times \overline{N},$$

则得

$$\bar{\bar{G}}(\overline{R}|\overline{R}') = -\frac{1}{k^2}\bar{\bar{I}}\delta(\overline{R} - \overline{R}') + \frac{1}{k^2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{q=0}^{\infty}\frac{k_\nu^2}{k_\nu^2 - k^2} \cdot \frac{\omega_0^2}{2(m+n+1)} \times \left(\overline{N}^{e}_{mnq}\overline{N}^{e'}_{mnq} + \overline{M}^{o}_{mnq}\overline{M}^{o'}_{mnq}\right), \tag{23}$$

其中 $\overline{M}, \overline{M}'$ 和 $\overline{N}, \overline{N}'$ 由式 (11) 和式 (12) 给出.

2 结语

本文利用近轴近似方法得到了有源加载情况下准光腔中的并矢格林函数. 这为求解腔内电磁场问题打下了基础.

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THE DYADIC GREEN'S FUNCTION IN AN ACTIVE LOADED QUASI-OPTICAL CAVITY*

HE YIJUN, YANG QUANRANG

(*State Key Laboratory of Millimeter Waves, Department of Radio Engineering, Southeast University, Nanjing, Jiangsu* 210018, *China*)

Abstract: The vector wave functions in an active loaded quasi-optical cavity were obtained with paraxial approximation. The dyadic Green's functions in the active loaded quasi-optical cavity were developed using the eigenvector wave function expansion. The integral solution of the electromagnetic field in the cavity was derived.

Key words: dyadic Green's function, quasi-optical cavity, active loaded.

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