

Weighted evidence combination based on distance of evidence and uncertainty measure

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Abstract: Dempster-Shafer evidence theory is an important tool in the fields of information fusion. However Dempster's rule of combination cannot efficiently handle highly conflicting evidence combination for it can arouse counter-intuitive behaviors. To deal with such a problem, a novel weighted average evidence combination approach is proposed. Not only the distance of evidence but also the uncertainty measure is utilized to determine the weights of the bodies of evidence. Based on the weighted averaged BOE and Dempster's rule of combination, the rational combination results can be obtained. The experimental results show that the method proposed can effectively handle conflicting evidence combination with better convergence.

Key words: evidence theory; sensor data fusion; distance function; uncertainty measure

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基于证据距离与不确定度的证据组合方法

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摘要: Dempster-Shafer 证据理论是信息融合领域中的一种重要的理论与方法. 然而在实际应用中, Dempster 证据组合规则无法有效处理高冲突证据组合问题, 往往引发反直观结果. 针对这一问题, 提出一种新的加权证据组合方法. 该方法同时利用证据距离和证据不确定度来生成权重进而修正待证组合证据, 并取得合理的组合结果. 实验结果表明所提方法具有更快的收敛速度, 能有效应对高冲突证据组合问题.

关键词: 证据理论; 传感器数据融合; 距离函数; 不确定度

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Introduction

Evidence theory^[1] is widely used in many fields of information fusion. In evidence theory, multiple inde-

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pendent bodies of evidence (BOEs) can be combined according to Dempster's rule of combination. However, it has counter-intuitive behaviors when highly conflicting BOEs are combined^[2]. Many approaches were proposed to resolve such a problem. In general, there are two main types of methodologies^[3]. One is to modify Dempster's rule of combination. The other is to pre-process the data (BOEs).

According to the first type of methodology, the counter-intuitive behaviors are imputed to the combination rule, especially the way to deal with the conflicting mass assignments. Various alternative rules^[3-5] were proposed. In the work of Lefevre^[6], a unified formulation for modified combination rules was proposed. In our previous work^[7], uncertainty measure is used to generate weights of BOEs to redistribute the conflicting mass assignments.

According to the second type of methodology, the counter-intuitive behaviors are imputed to the sensors or the BOEs obtained, for example, the evidence discounting^[1,8]. Murphy proposed a combination rule^[9] based on arithmetic average of the original BOEs. In Murphy's simple averaging approach, all BOEs seem equally important. However, in practice, it is not always reasonable. In our previous work, the evidence combination based on weighted average of evidences^[3,10] was proposed. The weights are generated based on the distance of evidence. It converges faster than Murphy's averaging method.

We prefer to modify the BOEs. In previous works, either the distance of evidence or the uncertainty measure has been used to determine the weights alone; however, the determination of weights should be more comprehensive. The uncertainty measure^[11] indicates the quality or clarity of the BOE and the distance of evidence represents the dissimilarity or the relation among different BOEs. If the two factors can be used jointly, better performance can be expected. According to such an idea, a novel weighted average evidence combination approach is proposed. The distance of evidence is used to generate the weights first, and then the weights are further modified by using uncertainty measure. Both the uncertainty measure of BOE and the distance of evidence can be derived based on BOEs,

thus no more extra priori knowledge is needed. An example on target recognition is provided, which show that the proposed approach is rational and effective.

1 Preliminary

1.1 Basics of evidence theory

In evidence theory^[1], the conception of basic probability assignment (BPA, also called mass function) $m:2^\Theta \rightarrow [0,1]$ is defined in equation (1). Θ is the frame of discernment (FOD).

$$\sum \{m(A) \mid A \subseteq \Theta\} = 1, m(\emptyset) = 0 \quad , \quad (1)$$

If $m(A) > 0$, A is called a focal element. The belief function (Bel) and the plausibility function (Pl) are defined as:

$$Bel(A) = \sum_{B \subseteq A} m(B) \quad , \quad (2)$$

$$pl(A) = \sum_{A \cap B \neq \emptyset} m(B) \quad , \quad (3)$$

When multiple independent BOEs are available, the combined evidence can be obtained based on Dempster's rule of combination as follows^[1]:

$$m(A) = \begin{cases} 0, & A = \emptyset \\ \frac{\sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j)}{1 - K}, & A \neq \emptyset \end{cases} \quad , \quad (4)$$

where $K = \sum_{A_i \cap B_j = \emptyset} m_1(A_i) m_2(B_j)$ represents the total conflicting or contradictory mass assignments. For Dempster's rule of combination, the conflicting mass assignments are discarded. Dempster's rule of combination is commutative and associative.

1.2 Counter-intuitive behaviors in Dempster's rule of combination

When there are high conflict among BOEs, counter-intuitive behaviors^[2] will emerge by using Dempster's rule of combination.

Zadeh's Example^[2]: Two doctors examine a patient and agree that he suffers from either meningitis (M), contusion (C) or brain tumor (T). Thus the FOD is $\Theta = \{M, C, T\}$. Assume that the doctors agree in their low expectation of a tumor, but disagree in likely cause and provide following diagnosis:

$$m_1(\{M\}) = 0.99, \quad m_1(\{T\}) = 0.01;$$

$$m_2(\{C\}) = 0.99, \quad m_2(\{T\}) = 0.01.$$

Based on Dempster's rule of combination, one gets the unexpected final conclusion: $m(\{T\}) = 1$. It

means that the patient definitely suffers from brain tumor. Obviously, this is a counter-intuitive result.

As aforementioned, there exist two major methodologies to deal with the highly conflicting evidence combination. We think that to modify the BOEs is more reasonable. Generally speaking, if the conflict is due to the malfunction of the sensors, modifying the combination rule to suppress the counter-intuitive result might not be proper.

2 A novel weighted evidence combination approach

Suppose that there are n BOEs $m_i, i = 1, \dots, n$, the pre-processing of the BOEs can be illustrated as:

$$m_{WAE} = \sum_{i=1}^n w_i \cdot m_i \quad , \quad (5)$$

where w_i is the corresponding weight of BOE m_i . In equation (5), each $w_i \cdot m_i$ can be considered as the discounted m_i , and m_{WAE} denotes the weighted averaged BOE of the original n BOEs. One can use Dempster's rule of combination to combine the weighted average of the BPA for $n - 1$ times to obtain the final result. n BOEs are weighted averaged according to all the available focal elements, respectively. How to get the appropriate weights?

If the priori knowledge is available, which can represent the quality or credibility of different BOEs, it can be used to generate the weights. But in fact, the priori knowledge is always hard to acquire. To be pragmatic, the weights should be generated by using the information provided by the BOEs themselves. Both distance of evidence and the uncertainty degree can be derived directly based on the BPAs of BOEs, and both of them individually has been successfully used to generate the weights of BOEs^[7, 10]. Distance of evidence represents the similarity (or dissimilarity) among BOEs. Uncertainty degree of a BOE describes its own clarity or quality. If such two kinds of factors can be used jointly, better combination results can be expected. It should be noted that the modification by using the distance of evidence is based on the assumption that "the truth lies in the majority", and the modification by using the uncertainty degree is based on the assumption that "the more clarity, the more credibility". In this paper, a novel weighted evidence combination

approach based on distance of evidence and uncertainty measure is proposed. The weights generation for averaging includes 2 steps: First, generate the weights according to the distance of evidence; Second, use the uncertainty measure to modify the weights generated in the first step.

2.1 Weight determination based on the distance of evidence

In Ref. [12], a distance of evidence is defined as:

$$d_J(m_i, m_j) = \sqrt{0.5(m_i - m_j) \mathbf{D}(m_i - m_j)} \quad , \quad (6)$$

In equation (6), m_i and m_j are two BPAs defined on FOD Θ . \mathbf{D} is a $2^n \times 2^n$ matrix. The element D in \mathbf{D} is defined as: $D(A, B) = |A \cap B| / |A \cup B|$, where $|\cdot|$ is cardinality.

The less the distance between two BOEs is, the more similarity between them is. According to our previous works^[3, 10], the similarity between m_i and m_j can be defined in equation (7):

$$Sim(m_i, m_j) = 1 - d_J(m_i, m_j) \quad , \quad (7)$$

The support degree of the BOE m_i is defined as:

$$Sup(m_i) = \sum_{j=1, j \neq i}^n Sim(m_i, m_j) \quad , \quad (8)$$

where n is the number of BOEs. Then the credibility of the BOE m_i is defined as follows:

$$Cred(m_i) = Sup(m_i) / \sum_{j=1}^n Sup(m_j) \quad , \quad (9)$$

In our previous work^[10], $Cred(m_i)$ are directly used to modify the BOEs according to equation (5). If the uncertainty degree can also be used on construct the weights, better combination results can be expected. The $Cred(m_i)$ derived by using distance of evidence are further modified by utilizing the uncertainty degree of BOEs as illustrated below.

2.2 Weight modification based on the uncertainty measure

Ambiguity measure^[11] (AM) defined in equation (10) is used as the uncertainty measure to modify the weights generated based on the distance of evidence.

$$AM(m) = - \sum_{\theta \in \Theta} BetP_m(\theta) \log_2(BetP_m(\theta)) \quad , \quad (10)$$

where $BetP_m(\theta) = \sum_{B \subseteq \Theta} m(B) / |B|$ is the pignistic probability^[13] proposed by Smets. AM is easy to compute and is relatively sensitive to the changes of evidence. It should be noted that AM has been criticized in the work of Klir^[14] for it can not satisfy the

sub-additivity for the joint-BPA. In our work, there is no problem of the joint-BPA, so the AM does work.

Suppose that some BOEs have relatively high support degree defined based on distance of evidence, i. e. they are relatively credible. If one of them has less uncertainty than the others, it should be a more credible BOE. This is because for such a BOE it is credible and at the same time it is relatively more lucid or clearer (less uncertainty). Thus it will be a high quality BOE, which is helpful for making right and solid decision. Such a BOE should have a larger weight. On the other hand, suppose that some BOEs have relatively low degree of support based on the distance of evidence, i. e. they are relatively incredible. If one of them has less uncertainty than the others, it should be more incredible. This is because for such a BOE it is incredible and at the same time it is relatively more exaggerated (less uncertainty). Such a BOE will be a poor quality BOE, which can more easily cause the wrong decision. Thus such a BOE should have a less value of weight.

According to such an idea, the weight modification can be implemented as follows.

(1) For all the BOEs: $m_i, i = 1, \dots, n$, calculate their corresponding $AM(m_i)$, respectively.

(2) Normalize the $AM(m_i)$ as follows:

$$Ent(m_i) = AM(m_i) / \sum_i AM(m_i) \quad , \quad (11)$$

(3) The modified weights are generated as follows:

$$Credm(m_i) = Cred(m_i) \times Ent(m_i)^{-\Delta Cred(m_i)} \quad , \quad (12)$$

where $Credm(m_i)$ is the weights generated according to equation (9) based on the distance of evidence. $Ent(m_i)$ is the normalized uncertainty degree of m_i obtained in equation (11). $\Delta Cred(m_i)$ is defined as:

$$\Delta Cred(m_i) = Cred(m_i) - \frac{1}{n} \sum_{j=1}^n Cred(m_j) \quad , \quad (13)$$

In equation (13) $\Delta Cred(m_i)$ is the difference between a BOE's credibility and the average credibility of all BOEs. The sign of $\Delta Cred(m_i)$ can be used to judge whether m_i is supported by the other BOEs or not. According to the idea of the weight modification proposed, if $Cred(m_i) > 0$ and m_i has relatively low value of $Ent(m_i)$, the effects of m_i should be strengthened. If $Cred(m_i) < 0$ and m_i has relatively low value

of $Ent(m_i)$, the effects of m_i should be suppressed. Such strengthening or suppressing can be implemented by using negative exponential function in equation (12) as analyzed below.

In Fig. 1, the negative exponential function with base number $k \in [0, 1]$ is illustrated. $Ent(m_i) \in [0, 1]$ is used as the base number (k_i in Fig. 1, where $i = 1, 2$.) of the negative exponential function. x -axis represents the value of $\Delta Cred(m_i)$; y -axis represents the value of $Ent(m_i)^{-\Delta Cred(m_i)}$.

As shown in Fig. 1, if $x > 0$ (i. e. $\Delta Cred(m_i) > 0$), then $y > 1$ (i. e. $Ent(m_i)^{-\Delta Cred(m_i)} > 1$). For the same $x > 0$, if $1 \geq k_1 > k_2 \geq 0$ then $(k_2)^{-x} > (k_1)^{-x}$. It means that the value of $Ent(m_i)^{-\Delta Cred(m_i)}$ will increase with the growth of $\Delta Cred(m_i)$ and with the decrease of $Ent(m_i)$. If $x < 0$ (i. e. $\Delta Cred(m_i) < 0$), then $y < 1$ (i. e. $Ent(m_i)^{-\Delta Cred(m_i)} < 1$). For the same $x < 0$, if $1 \geq k_1 > k_2 \geq 0$ then $(k_2)^{-x} < (k_1)^{-x}$. It means that the value of $Ent(m_i)^{-\Delta Cred(m_i)}$ will decrease with the decrease of $\Delta Cred(m_i)$ and with the decrease of $Ent(m_i)$.

Based on the analysis above and equation (12), it can be seen that there exists $Credm(m_i) > Cred(m_i)$ for m_i with $\Delta Cred(m_i) > 0$ due to $Ent(m_i)^{-\Delta Cred(m_i)} > 1$; meanwhile there exists $Credm(m_i) < Cred(m_i)$ for m_i with $\Delta Cred(m_i) < 0$ due to $Ent(m_i)^{-\Delta Cred(m_i)} < 1$. And the less the uncertainty degree $Ent(m_i)$ is, the more significant difference between $Credm(m_i)$ and $Cred(m_i)$ is. This means that the effect of the more credible evidences are strengthened and the effect of the more incredible evidence are suppressed based on equation (12) by further using uncertainty degree AM.

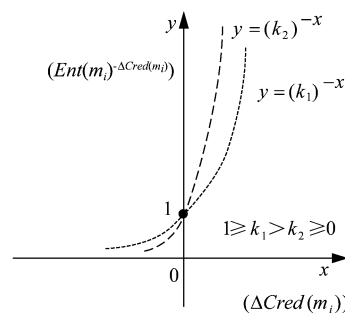


Fig. 1 Negative exponential curve (base number $\in [0, 1]$)
图 1 负指数曲线 (底数 $\in [0, 1]$)

(4) Normalize all the $Credm(m_i)$ as follows:

$$Credmn(m_i) = Credm(m_i) / \sum_i Credm(m_i) \quad , \quad (14)$$

Based on the $Credmn(m_i)$ derived and equation (5), the weighted averaged BOE is derived as follows:

$$m_{WAE} = \sum_{i=1}^n (Credmn(m_i) \times m_i) \quad , \quad (15)$$

In the final, m_{WAE} derived in equation (15) are combined $n - 1$ times by using Dempster's rule of combination, the combined BOE can be obtained.

3 Example

In a multisensor-based target recognition system, there are totally three types of targets: $\Theta = \{A, B, C\}$. Suppose the real fault type is A . There are five different sensors including CCD (S1), audio sensor system (S2), infrared system (S3), Radar (S4), and ESM (S5). From five different sensors, the system has acquired five BOEs listed as follows:

$$S1: m_1(A) = 0.41, m_1(B) = 0.29, m_1(C) = 0.3;$$

$$S2: m_2(A) = 0, m_2(B) = 0.9, m_2(C) = 0.1;$$

$$S3: m_3(A) = 0.58, m_3(B) = 0.07, m_3(AC) = 0.35;$$

$$S4: m_4(A) = 0.55, m_4(B) = 0.1, m_4(AC) = 0.35;$$

$$S5: m_5(A) = 0.6, m_5(B) = 0.1, m_5(AC) = 0.3.$$

In this example, for the acquirement of the incredible BOE m_2 , which may be caused by the errors of operator or the flaws of the sensor itself.

The results derived based on different combination rules are listed in Table 1. As illustrated in Table 1, when conflicting BOEs are acquired, Dempster's rule

of combination outcomes counter-intuitive results. When more types of sensors are available, i. e. when more BOEs are available, Murphy's simple averaging, Deng's weighted averaging and the weighted averaging proposed in this paper all provide reasonable results. As illustrated in Table 1, the performance of convergence of the proposed method is better than that of Murphy's simple averaging and Deng's simple averaging (our previous work). The reason is that our proposed method can strengthen the effect of credible evidence further and at the same time weaken the effect of incredible evidence further. Furthermore, it should be noted that there is no unified or standard evaluation criterion for the evidence combination. In practical applications, it can be evaluated from two aspects below. The first one is whether the combination result is accordant to the intuitive and logical reasoning or not. The other one is whether the uncertainty can be decreased after the evidence combination. From such two aspects, our proposed approach is also rational and effective.

4 Conclusions

Dempster's rule of combination can out-come counter-intuitive results when the different BOEs to be combined are highly conflicting. The proposed weighted averaging approach by jointly using the distance of evidence and the uncertainty measure preserves all the

Table 1 Evidence combination results based on different combination rules

表 1 基于不同组合规则的证据组合结果

Approach	Combination results			
	m_1, m_2	m_1, m_2, m_3	$m_1, m_2, m_3, m_4,$	m_1, m_2, m_3, m_4, m_5
Dempster's rule ^[11]	$m(A) = 0$	$m(A) = 0$	$m(A) = 0$	$m(A) = 0$
	$m(B) = 0.8969$	$m(B) = 0.6575$	$m(B) = 0.3321$	$m(B) = 0.1422$
	$m(C) = 0.1031$	$m(C) = 0.3425$	$m(C) = 0.6679$	$m(C) = 0.8578$
Murphy's simple average ^[14]	$m(A) = 0.0964$	$m(A) = 0.4619$	$m(A) = 0.8362$	$m(A) = 0.9620$
	$m(B) = 0.8119$	$m(B) = 0.4497$	$m(B) = 0.1147$	$m(B) = 0.0210$
	$m(C) = 0.0917$	$m(C) = 0.0794$	$m(C) = 0.0410$	$m(C) = 0.0138$
	$m(AC) = 0$	$m(AC) = 0.0090$	$m(AC) = 0.0081$	$m(AC) = 0.0032$
Deng's weighted average ^[15]	$m(A) = 0.0964$	$m(A) = 0.4974$	$m(A) = 0.9089$	$m(A) = 0.9820$
	$m(B) = 0.8119$	$m(B) = 0.4054$	$m(B) = 0.0444$	$m(B) = 0.0039$
	$m(C) = 0.0917$	$m(C) = 0.0888$	$m(C) = 0.0379$	$m(C) = 0.0107$
	$m(AC) = 0$	$m(AC) = 0.0084$	$m(AC) = 0.0089$	$m(AC) = 0.0034$
This paper	$m(A) = 0.0964$	$m(A) = 0.5188$	$m(A) = 0.9246$	$m(A) = 0.9844$
	$m(B) = 0.8119$	$m(B) = 0.3802$	$m(B) = 0.0300$	$m(B) = 0.0023$
	$m(C) = 0.0917$	$m(C) = 0.0926$	$m(C) = 0.0362$	$m(C) = 0.0099$
	$m(AC) = 0$	$m(AC) = 0.0084$	$m(AC) = 0.0092$	$m(AC) = 0.0034$

特异材料光轴与 z 轴夹角 α 、特异材料薄层厚度 d 对 GH 位移的影响. 研究发现,适当选取特异材料的电磁属性、薄层厚度 d 、夹角 α 以及入射角 φ ,可获得正的或负的具有大幅度的 GH 位移. 其中,对于 cutoff, anti-cutoff 和 never cutoff 型特异材料,薄层厚度 d 和夹角 α 对 GH 位移都有着显著的影响,而对于 always cutoff 型特异材料,不论以何入射角入射,反射波总存在负的 GH 位移. 通过对特异材料结构中 GH 位移的研究,将有利于特异材料在微波或光学系统中的应用.

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desirable properties of the Murphy's averaging and Deng's weighted averaging. In addition, the proposed method can efficiently handle conflicting evidence with better performance of convergence.

In future work, more factors will be analyzed and used in establishing the weight to construct more powerful evidence combination approaches.

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